GRAPHICAL FLOOD ROUTING AND RELATED MECHANICAL ANALOGS

By Max A. Kohler

The dampening effect of valley storage on a wave moving through a river reach is a familiar phenomenon and hydrologic literature abounds with descriptions of routing methods--analytical, graphical, electric analogs and mechanical devices[1,2,3]. A similar dampening is apparent if one compares the rear-wheel track of a moving vehicle with that made by a front wheel, although closer examination will reveal that the two effects are not alike in all respects for a vehicle with fixed wheel base. Nonetheless, study of the two pnenomena reveals some rather interesting results pointing the way for development of mechanical analogs and simplified graphical solutions to flood routing problems.

Linear Storage-Discharge Function

Let us first consider the case of a linear storage discharge relation, i.e.,

$$(1) \quad S = KQ$$

where S is the reached storage, Q is the reach outflow, and K is a constant having the units of time. Figure 1 shows schematically the requirements for an analog which will satisfy this function. In this diagram, the time scale for outflow is shifted K time units to the left of the inflow scale and the tongue connecting the inflow tracer to the outflow pen is adjustable so as to maintain the time displacement between the two at K units. The periphery of the wheels carrying the outflow pen would be knife-edged and possibly saw-toothed to eliminate lateral slippage. Under these circumstances, it will be seen that

$$(2) \quad \frac{dQ}{dt} = \frac{I - Q}{K} = \frac{1}{K} \frac{dS}{dt}$$

which is equivalent to Equation 1.

Some thought has been given to the design of a mechanical analog which would satisfy Eq. 1 and still trace the outflow on the same time scale (and chart) as the original inflow hydrograph. While this possibly can be done, it is much simpler to design one which will trace the outflow at the proper time-scale, but on a separate chart. Figure 2 shows such an analog which can be fabricated primarily from plastic sheets. If the uppermost sheet of plastic (embodying the inflow tracer and the outflow pen) is mounted rigidly to the head of a drafting machine, the need for the other sheets is eliminated.

A simple, rapid graphical solution for Eq. 1 is also obvious from Fig. 1, and the only aid required is a straight-edge[3]. If the outflow up to any initial time is plotted K units to the left of the inflow hydrograph to be routed, a straight-edge connecting simultaneous values of I and Q has the slope dQ/dt, and the outflow can be extended for a reasonable time with this slope. Moving to successive later times, the inflow hydrograph is routed segment by segment. The routing period (i.e., the length with which the outflow curve is extended in one step) can be varied at random, depending upon the rate of change dQ/dt. Moreover, the outflow and inflow time scales can be made to coincide by placing the straight-edge so as to connect the outflow with simultaneous inflow lagged K time units (see Fig. 3). In this case, it is preferable to have a straight-edge calibrated in time units to assist in lagging the inflow by K units. This method[3] of routing is somewhat analogous to that described by Nash and Farrell[2], but provides a much smoother outflow hydrograph in less time.

Muskingum Storage Function

The Muskingum method of routing developed by McCarthy[1] is based on the storage function

$$(d)\quad S = K\left[xI + (1 - x)Q\right]$$

where x is visualized as a weighting factor which takes into account wedge storage,

Some thought has been given to the design of a mechanical analog which would multiply Eq. 1 and graph brown the airflow on the same time scale (and above) as the original airflow hydrograph. While this possibly can be done, it is much simpler to design one which will trace the airflow on the proper time-scale, as on a separate chart. Figure 2 shows such an analog which can be fabricated readily from plastic sheets. If the upper-most sheet of plastic (embodying the inflow trace and the airflow pen) is mounted rigidly on the head of a drafting machine, the head for the other sheets is eliminated.

A simple, rigid graphical solution for Eq. 1 is also obvious from Fig. 1, and the only and required is a straight-edge[3]. If the airflow up to any initial time is plotted X units to the left of the inflow hydrograph to be routed, a straight-edge connecting simultaneous values of I and Q has the slope dQ/dt, and the airflow can be extended for a reasonable time with this slope. Moving to successive later times, the inflow hydrograph is routed segment by segment. The routing period (i.e., the length with which the outflow curve is extended in one step) can be varied at random, depending upon the rate of change dQ/dt. Moreover, the airflow and inflow time scales can be made to coincide by placing the straight-edge so as to change the airflow with simultaneous inflow lagged X time units (see Fig. 3). In this case, it is preferable to have a straight-edge calibrated in time units to assist in lagging the inflow by X units. This method[3] of routing is somewhat analogous to that described by Zoch and Parkin[2], but provides a much smoother airflow hydrograph in less time.

Maximum Storage Function

The maximum method of routing developed by Matthews[4] is based on the storage function:

$$(4)\quad S = X\left[\alpha I + (1 - \alpha)Q\right]$$

where α is characterized as a weighting factor which takes into account water storage.

and all other symbols are as previously defined. Figure 4 illustrates schematically the requirements for a mechanical analog satisfying Eq. 3. If the tongue is jointed as shown; if the time axis of the inflow end of the tongue is at all times equal to Kx and that of the entire tongue is equal to K; and if the inflow portion of the tongue is always pivoted so as to have the slope dI/dt, then

$$(4)\quad I - Q = \frac{dS}{dt} = K(1-x)\frac{dQ}{dt} + Kx\frac{dI}{dt}$$

The changes required to make the analog of Fig. 2 adaptable to the more general Muskingum equation are obvious, although it becomes relatively much more complex and requires that the operator follow both the inflow discharge and the slope of the inflow hydrograph. Similarly, graphical routing can be accomplished with the aid of a jointed straight-edge of the general character shown in Fig. 5. Actually, however, experience has shown that the Muskingum equation provides no more flexibility than does Eq. 1 employed in connection with a lagged inflow. Since both the analog of Fig. 2 and the corresponding graphical solution (Fig.3) can readily incorporate fixed lag L (by a horizontal shift of the outflow chart in the case of the analog), there is little to recommend use of the Muskingum function.

Variable Lag and Storage Factor

There are numerous reaches for which neither the Muskingum nor the lag-and-route (constant K and L) techniques provide satisfactory results. In fact, it is quite generally found that the results of these approaches can be improved upon if L and K are permitted to vary with flow in the reach. It can be shown that inflow and outflow volumes do not check when K is assumed to be a function of inflow and, therefore, K must be related to outflow. Lag can be made a function of either inflow or outflow. That is, the inflow hydrograph can be lagged as a function of the inflow discharge and the result routed with a variable K, or the fnflow can

and all other symbols are as previously defined. Figure 4 illustrates schematically the requirements for a mechanical analog satisfying Eq. 3. If the torque is jointed as shown, if the time axis of the inflow end of the torque is at all times equal to Kx and that of the entire torque is equal to K_1 and if the inflow portion of the torque is always pivoted so as to have the slope dI/dt, then

$$(4)\quad I - q = \frac{dq}{dt} = K(1-x)\frac{dq}{dt} + Kx\frac{dI}{dt}$$

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be routed with K as a function of outflow and the resulting hydrograph lagged. While reasonably comparable results can be obtained in either manner, practical considerations, both in development and application, favor the use of $L = f(I)$.

It is not the purpose of this paper to describe in detail the actual development of routing procedures, but as a brief outline for the variable K and L technique will aid in following the subsequent discussion. If storage is a function of outflow only, then it follows that the peaks and troughs of the routed outflow fall on the inflow hydrograph (i.e., when $\frac{dQ}{dt} = 0$, $I = Q$). Therefore, a good approximation to $L = f(I)$ can obviously be derived by analysis of a series of inflow-outflow hydrographs (Fig.6) to determine the required values of L, and plotting such values vs corresponding values of I.

Replotting the inflow hydrographs lagged in accordance with $L = f(I)$ as determined above, the values of K can be obtained as shown in Fig. 7. Placing a straight-edge tangent to the outflow curve (at any point where $\frac{dQ}{dt} \longrightarrow 0$), K is read as the time difference between the lagged inflow (at time of selected outflow) and the straight-edge (Eq.2). Having determined K at each of a number of points for selected rises, the relation $K = f(Q)$ can be derived by plotting K vs Q.

Relations for K and L developed in the manner described are usually highly satisfactory. Nevertheless, comparison of routed and observed hydrographs may indicate improved results through minor adjustments. The analysis for determination of K and L may show there is little to be gained by permitting these factors to vary with flow, and in this case the simpler solution is to be preferred. Limited experience has shown that $K = f(q)$ and $L = f(I)$ are quite similar for a particular reach--if L or K varies appreciably with flow, then both vary in a similar fashion.

be tested with $\underline{K}$ as a function of outflow and the resulting hydrograph lagged. While theoretically comparable results can be obtained in either manner, practical considerations, both in development and application, favor the use of $L = f(I)$.

It is not the purpose of this paper to describe in detail the actual development of routing procedures, but rather a brief outline for the variables $\underline{K}$ and $\underline{L}$ seemingly will aid in following the subsequent discussion. If storage is a function of outflow only, then it follows that the peaks and troughs of the routed outflow fall on the inflow hydrograph (i.e., when $\frac{dS}{dt} = 0$, $I = O$). Therefore, a good approximation to $L = f(I)$ can obviously be derived by analysis of a series of inflow outflow hydrographs (Fig.4) to determine the regulated values of $\underline{L}$, and plotting such values vs corresponding values of $\underline{I}$.

Regulating the inflow hydrographs lagged in accordance with $L = f(I)$ as determined above, the values of $\underline{K}$ can be obtained as shown in Fig. 7. Placing a straight-edge tangent to the outflow curve (at any point where $\frac{dO}{dt} \longrightarrow 0$), $\underline{K}$ is read as the time difference between the lagged inflow (at time of maximum outflow) and the straight-edge (Fig.5). Having determined $\underline{K}$ at each of a number of points for maximum rises, the relation $K = f(O)$ can be derived by plotting $\underline{K}$ vs $\underline{O}$.

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Graphical routing with variable K and L, although more complex than when using constant factors, is more rapid than many less flexible methods. It is greatly facilitated by the aid of a transparent template (Fig. 8), with the central section removed. The left-hand edge of the cutout section represents L = f(I), and the right-hand edge represents K = f(Q), both as measured from the vertical, central base line. The template is moved horizontally along a straight-edge to determine a segment of the outflow hydrograph centered at the base line. Use of the template is illustrated in Fig. 9. Intersection of the inflow hydrograph with the left-hand edge of the template represents lagged inflow at time indicated by the base line. Eye extrapolation of the outflow curve to the base line determines the value of K on the right-hand edge of the template. The point "A" thus obtained is connected by straight-edge to the previously-determined segment of the outflow hydrograph and a reasonable (depending upon the rate of change of slope of the hydrograph) extension to the curve is constructed in a manner similar to that shown in Fig. 3. This process is repeated until the routing operation is completed.

Figures 6 thru 9 are based on data for the Independence-Lenopah reach of the Verdigris River (Kansas-Oklahoma). The template showing variations of L and K (Fig. 8 and 9) was developed from data on numerous rises in addition to those shown in Figs. 6 and 7.

The mechanical analog for accomplishing this form of routing must naturally be more complex than that illustrated in Fig. 2. The time difference between the inflow tracer and the outflow pen must be varied in accordance with L = f(I), while the time difference between the outflow pen and the anchor point for the outflow tongue must follow K = f(Q). Modification for variable lag is reasonably simple, but variations in K must be geared to the outflow pen and this presents quite a different problem. The required mechanism can be easily designed, but vertical movement of the outflow pen is forced to "work" the mechanism, which would tend to

Graphical routing with variable $\underline{K}$ and $\underline{L}$, although more complex than using constant factors, is more rapid than analogous functional methods. It is greatly facilitated by the use of a transparent template (Fig. 8), with two central section removed. The left-hand edge of the outer section represents $L = f(z)$, and the right-hand edge represents $K = f(y)$, both as measured from the vertical, central base line. The template is moved horizontally along a straight-edge to determine a segment of the outflow hydrograph centered on the base line. The use of the template is illustrated in Fig. 9. Intersection of the inflow hydrograph with the left-hand edge of the template represents lagged inflow at time indicated by the base line. The extrapolation of the outflow curve to the base line determines the value of $\underline{K}$ on the right-hand edge of the template. The point "A" thus obtained is connected by straight-edge to the previously-determined segment of the outflow hydrograph and a tangent (depending upon the rate of change of slope of the hydrograph) extension to the curve is constructed in a manner similar to that shown in Fig. 3. This process is repeated until the routing operation is completed.

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induce side thrust on the wheels and thus possibly distort the outflow hydrograph. Even so, such a design may be feasible.

Although a more difficult task, it is believed an operator could soon acquire skill to follow the inflow hydrograph and simultaneously make mechanical adjustment for variations in K. Figure 10 shows the basic features of an analog designed on this basis. By lateral and vertical movement of the left-hand knob, the inflow pointer is made to trace the inflow hydrograph and rotation of the knob makes indicated adjustments of K. A right-hand knob is provided to assist in lateral (time) motion. Use of the analogs illustrated is admittedly more convenient for a left-handed operator (the author happens to be), and it may be advantageous to reverse the mechanism. Certain complications arise by virtue of the fact that the hydrographs must be traced from left to right, however, which tends to defeat the purpose.

Summary and Conclusions

Graphical techniques for routing directly on the plotted hydrograph charts have been presented for (1) the case of simple, linear storage-discharge function, (2) the case of linear storage function in conjunction with constant lag (translation), and (3) the case of both lag and the storage factor varying as complex functions of flow in the reach. The third category represents what is believed to be one of the most flexible methods of routing yet devised. Amazingly reliable results have been obtained for reaches which are incompatible with storage functions of the Muskingum and other recognized methods of routing. The time required for procedure development compares favorably with any other known technique. Moreover, the graphical solutions illustrated are operationally rapid and reliable.

The basic features of mechanical analogs (comparable to the graphical solutions described) have also been discussed. Although the details of design are quite well established for the simpler analog (constant K and L), neither type

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The author is indebted to Messrs. T.J. Nordenson and W.K. Fox for their suggestions and assistance in editing the manuscript.

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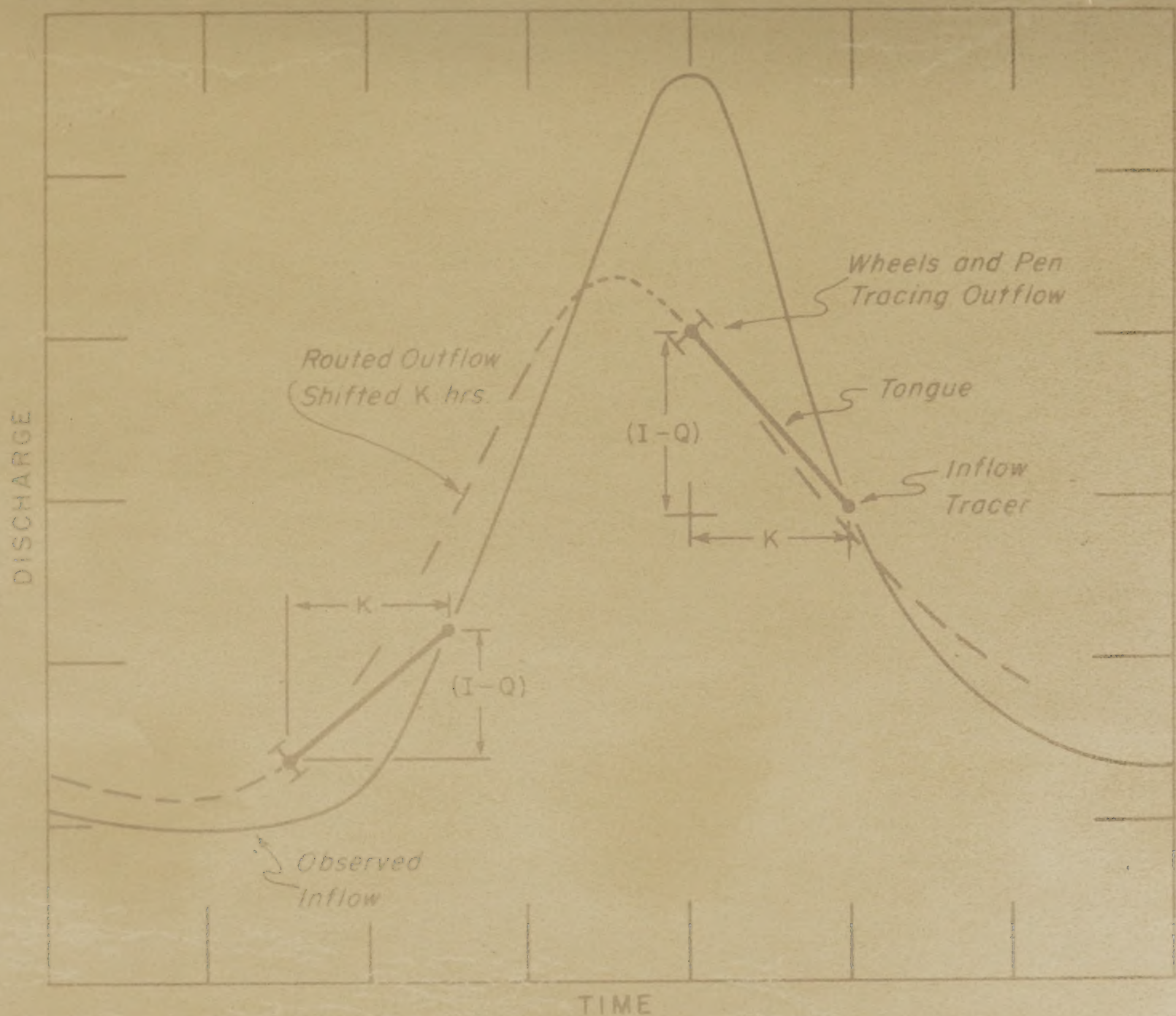

Fig. 1 Schematic diagram of analog for linear storage-discharge function.

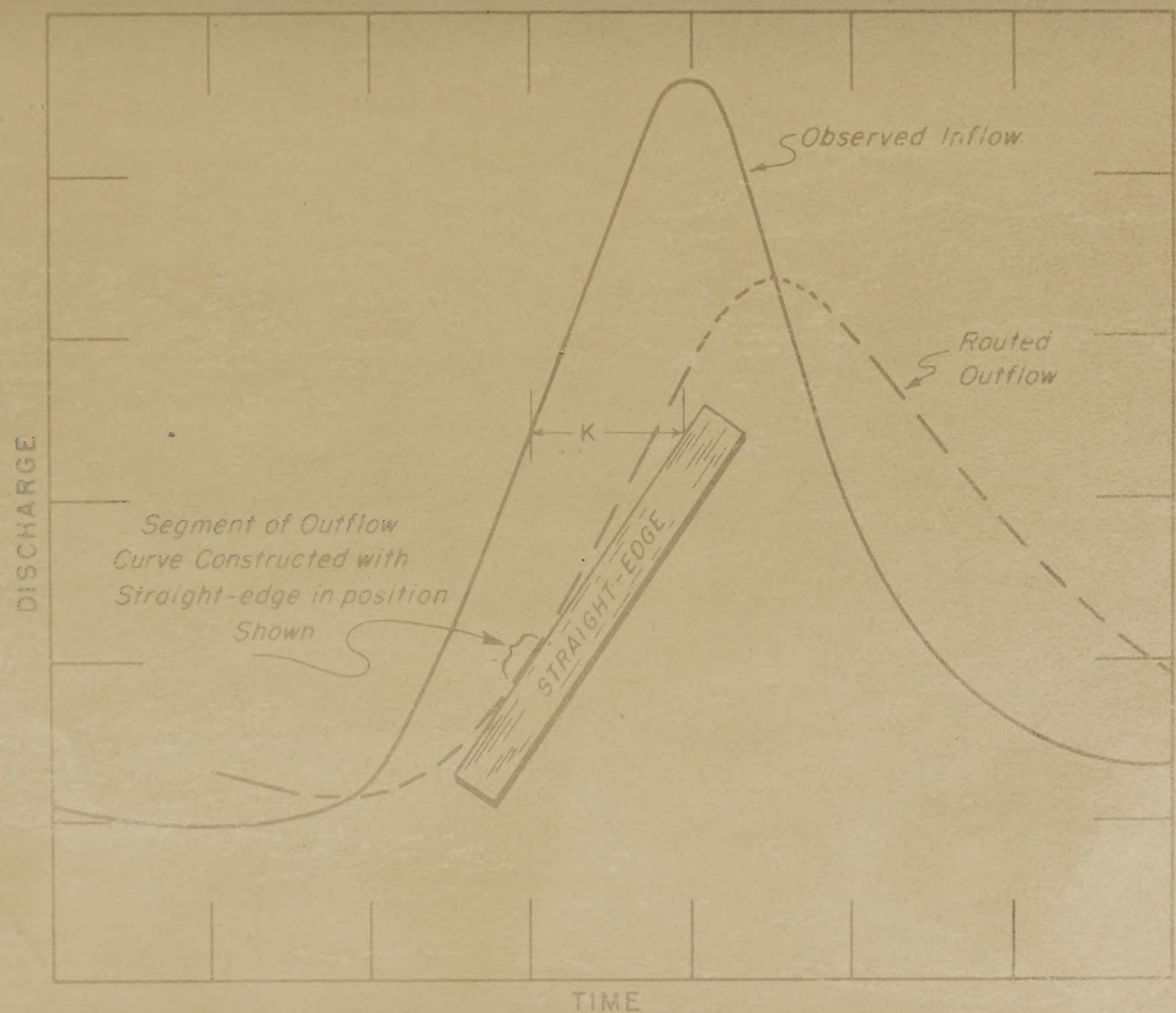

Fig. 3. Graphical routing with linear storage-discharge function.

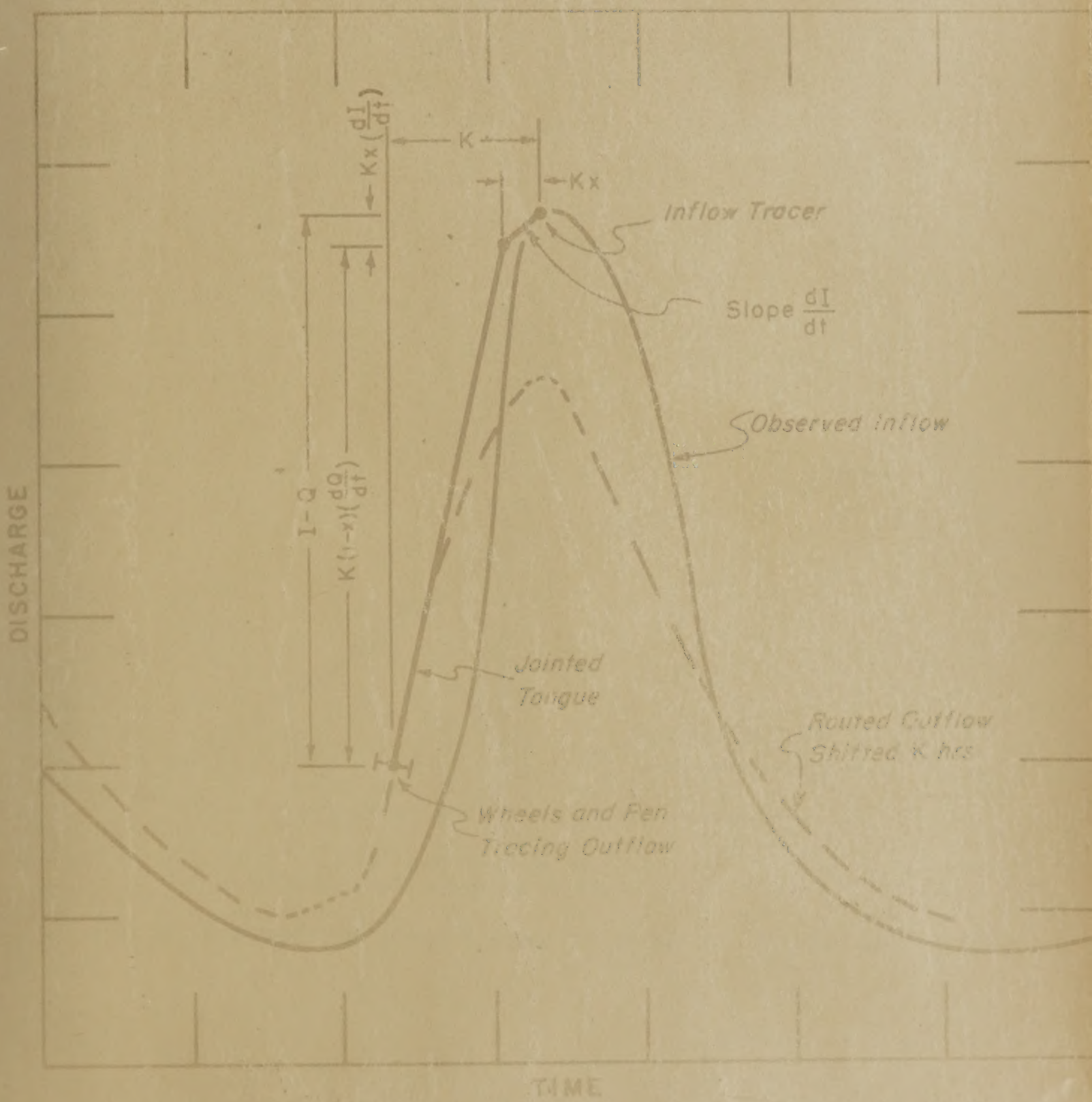

Fig. 4. Schematic diagram of analog for Muskingum routing.

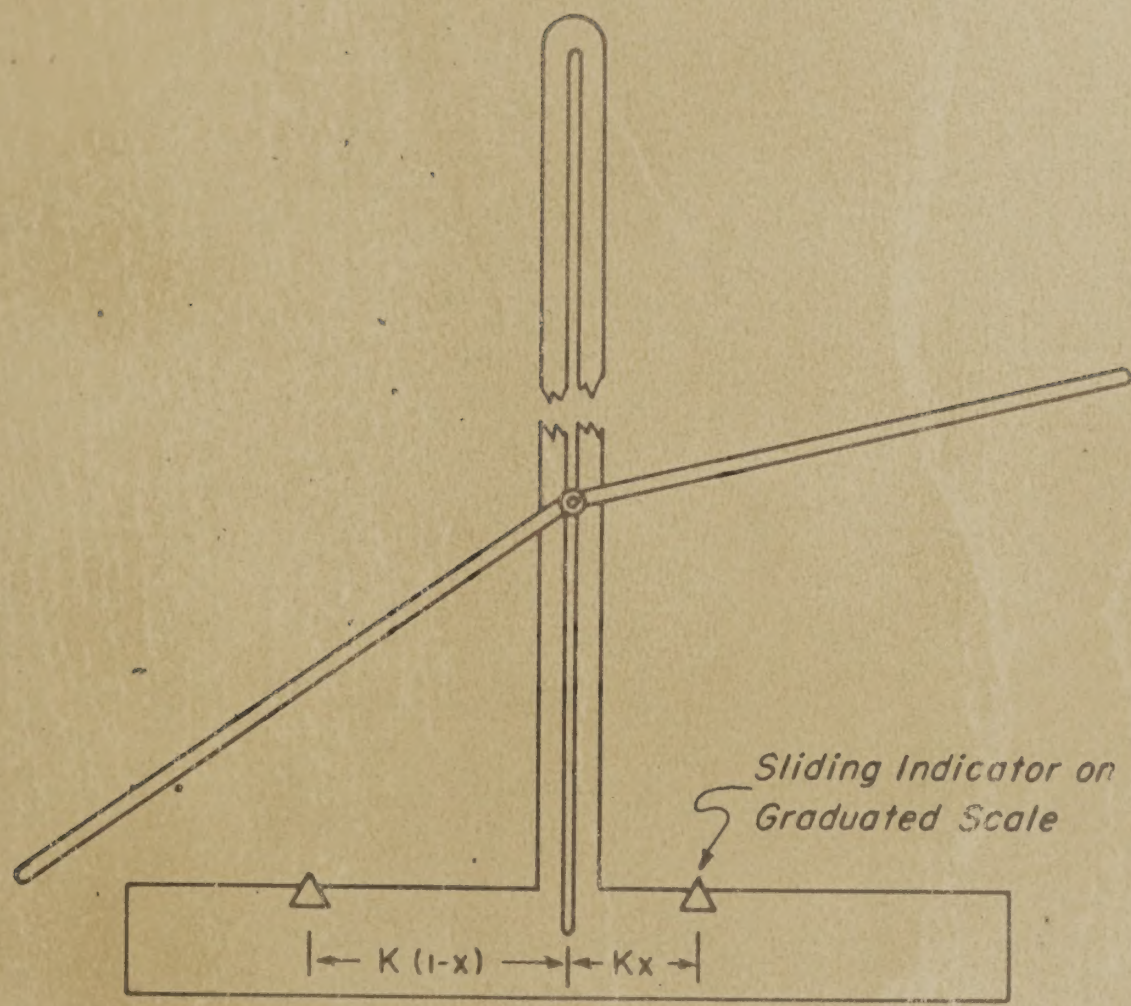

Fig. 5. Transparent plastic aide for graphical Muskingum routing.

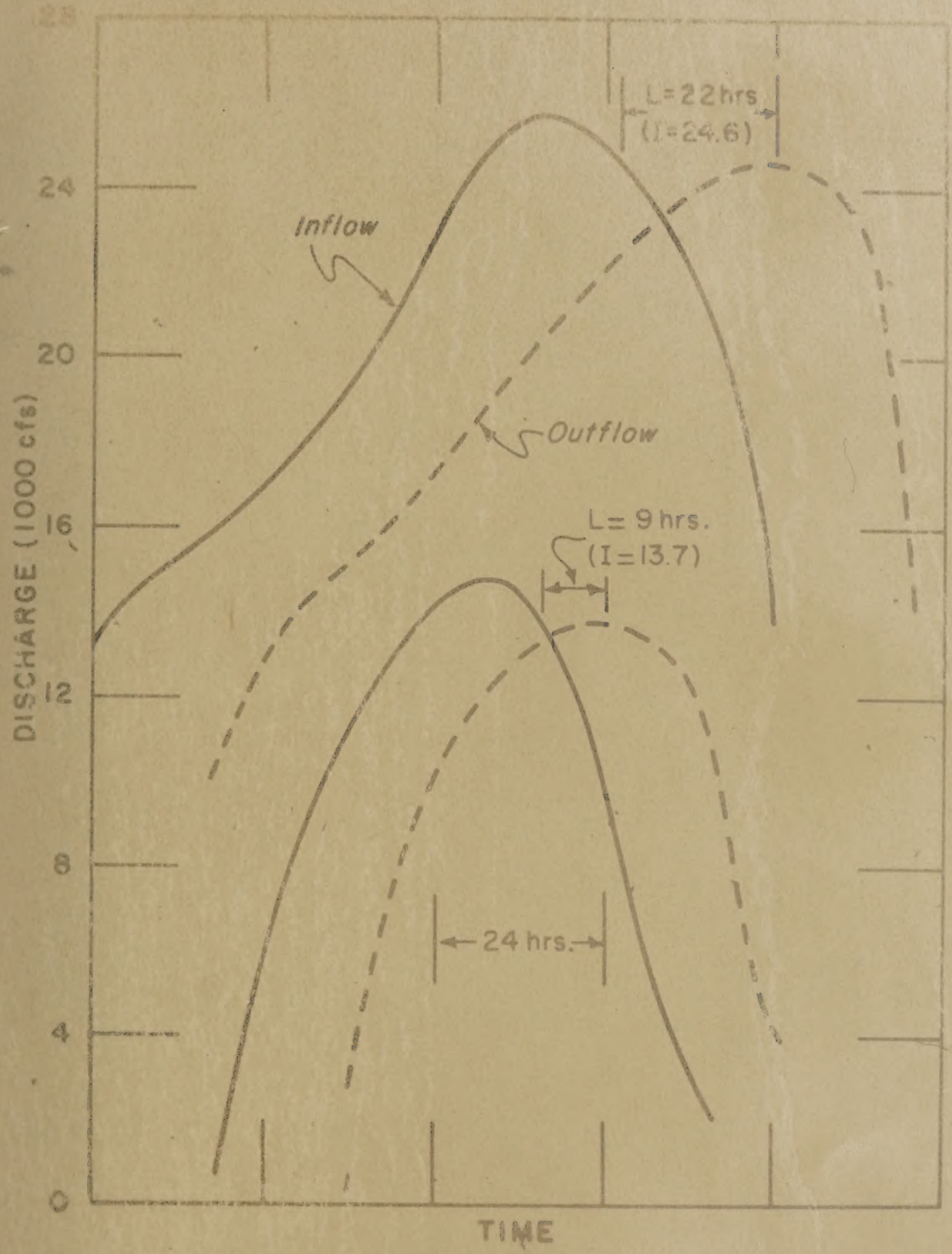

Fig. 6. Estimation of required lag (L).

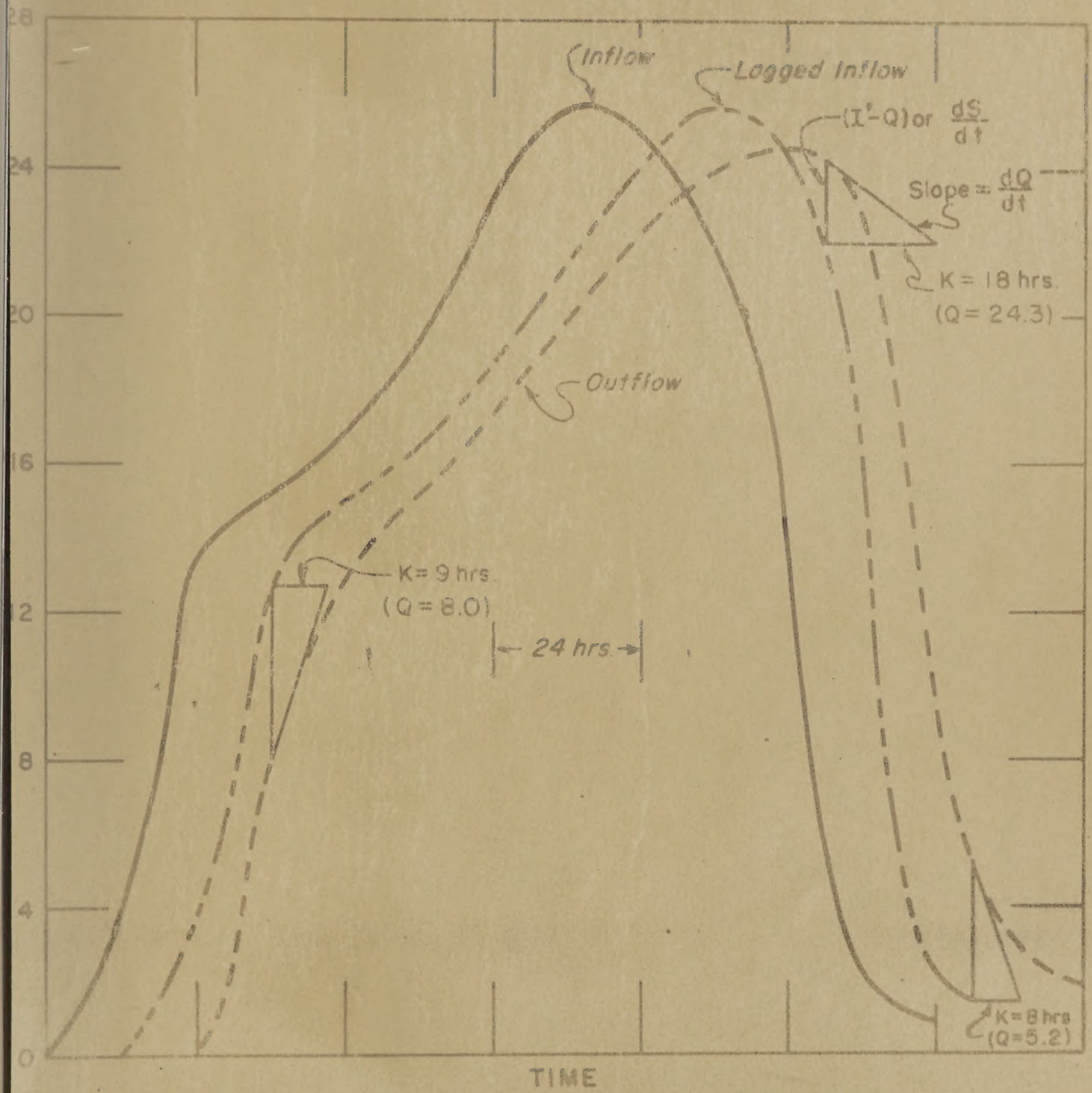

Fig. 7. Estimation of required storage factor (K).

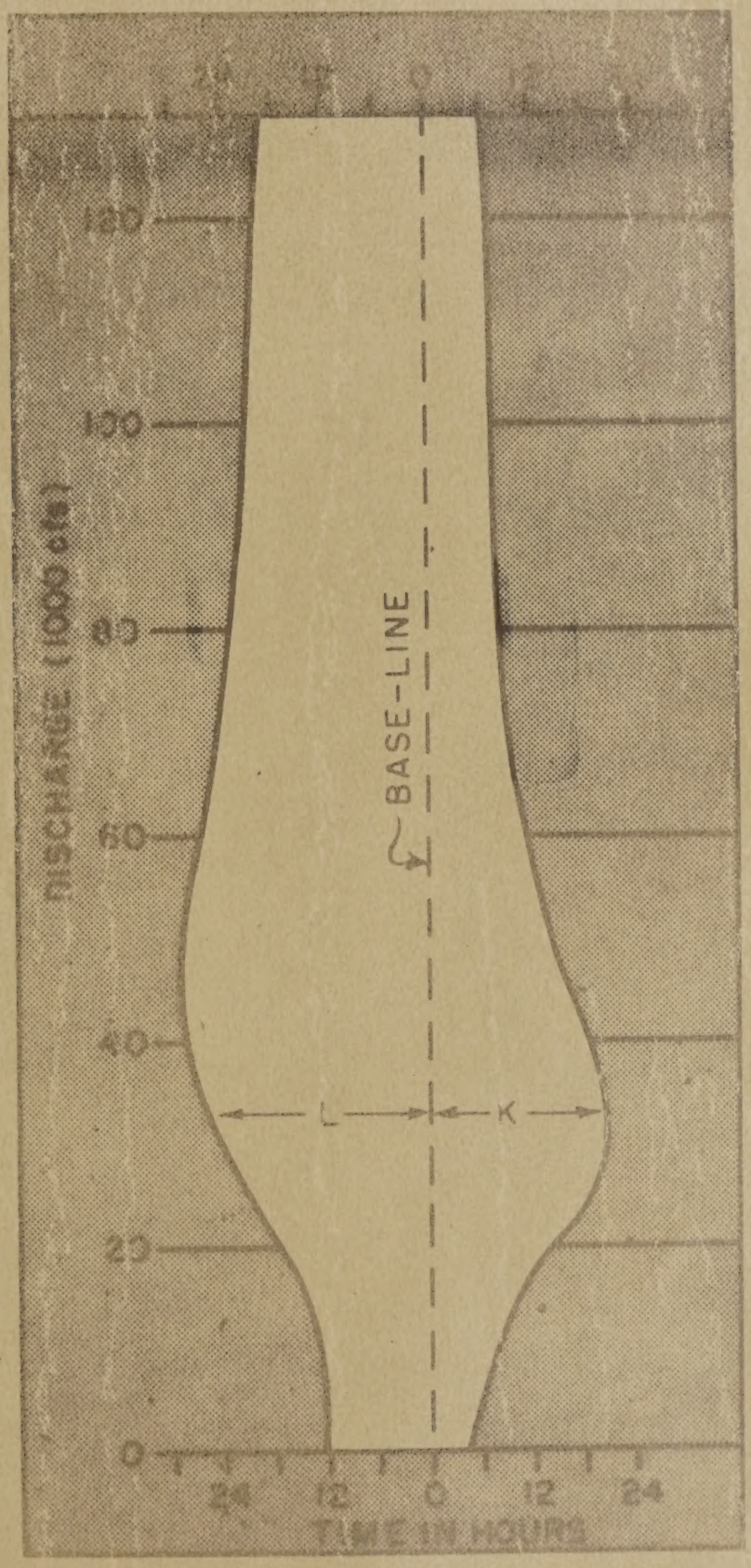

Fig. 8. Template for routing with variable lag and storage factor

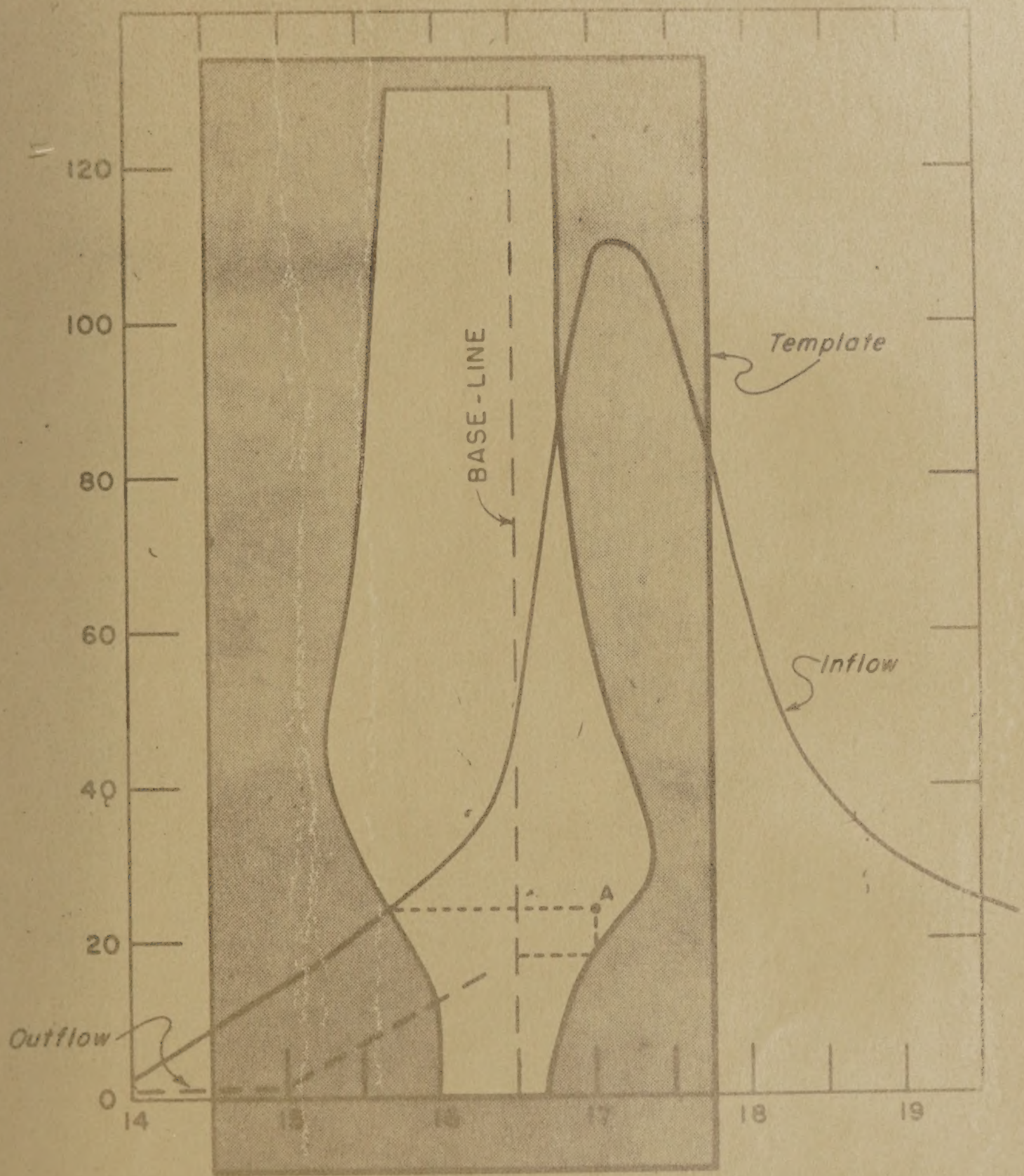

Fig. 9. Illustrating use of template shown in Fig. 8